Dual-polarization MSBL-based Beamforming for GNSS Multipath Mitigation

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***Abstract*—Multipath environment may enormously hamper the tracking performance. To enhance multipath mitigation, we propose a novel method based on multi-dimensional processing and multiple sparse Bayesian learning (MSBL). Our method utilizes sparsity in the spatial domain and signal characteristics in the polarization domain. Then we derive a joint off-grid angle and polarization MSBL-based beamformer. The LOS time delay is estimated from the proposed beamformer output with a maximum likelihood estimator. Simulation results demonstrate that our approach achieves close-to-optimal performance compared to the Cramér-Rao lower bound and outperforms an equivalent single-polarization beamformer with multipath signals.**

Keywords—Multipath mitigation; Multiple sparse Bayesian learning; Off-grid estimation; Dual-polarization antenna array

I. Introduction (*Heading 1*)

Significant efforts have been made to mitigate multipath effects in urban environments, as they can introduce bias up to meters in GNSS receivers. Traditionally, multipath mitigation methods have focused on either space [1] or polarization domains. However, they are challenging unless there is a significant separation between the line-of-sight (LOS) and the multipath signals in the corresponding dimension. Joint space and polarization processing is a promising approach. By exploiting the polarization diversity in addition to the spatial diversity, various beamformers based on angle and polarization estimates of multipath signals [2],[3], have been designed.

In this paper, since multiple sparse Bayesian learning (MSBL) always has the sparsest global minima [4], we propose a joint angle and polarization multiple sparse Bayesian learning (JAPMSBL) method to improve multipath mitigation, as well as especially when a high degree of coherence exists between the LOS and the multipath signals.

II. Signal Model

A. GNSS Multipath Signal Model

The baseband signal received by M_θ -pair element dual-polarization array (DPA) can be expressed as

$$\begin{aligned}\boldsymbol{y}(t)&=\sum_{k=0}^{K}\begin{bmatrix}\boldsymbol{x}_{Rc,k}(t)+\boldsymbol{x}_{Rx,k}(t)\\ \boldsymbol{x}_{Lx,k}(t)+\boldsymbol{x}_{Lc,k}(t)\end{bmatrix}+\boldsymbol{e}(t)\\ &=\sum_{k=0}^{K}\begin{bmatrix}\gamma_k\left(\lambda_{R,k}\boldsymbol{a}_{Rc}(\theta_k)+\lambda_{L,k}\boldsymbol{a}_{Rx}(\theta_k)\right)c(t-\tau_k)\\ \gamma_k\left(\lambda_{R,k}\boldsymbol{a}_{Lx}(\theta_k)+\lambda_{L,k}\boldsymbol{a}_{Lc}(\theta_k)\right)c(t-\tau_k)\end{bmatrix}+\boldsymbol{e}(t)\end{aligned} \tag{1}$$

where the upper part includes the k th co-polar RHCP signal $\boldsymbol{x}_{Rc,k}(t)$ and cross-polar LHCP signal $\boldsymbol{x}_{Rx,k}(t)$ So does the lower part. The right and left co-polar and the cross-talk steering vectors are denoted as $\boldsymbol{a}_{Rc}(\theta_k),\boldsymbol{a}_{Lc}(\theta_k)$ and $\boldsymbol{a}_{Rx}(\theta_k),\boldsymbol{a}_{Lx}(\theta_k)$ of the angle θ_k . The k th signal has a complex amplitude of γ_k , with right and left reflection coefficients $\lambda_{R,k}$ and $\lambda_{L,k}$. The k th known waveform is $c(t)$ with time delay τ_k. The noise is modeled as $\boldsymbol{n}(t)$.

Collecting N samples and representing the signal model (1) in a matrix form

$$\boldsymbol{y}(n)=\boldsymbol{A}(\boldsymbol{\theta})\operatorname{diag}(\boldsymbol{\lambda})\boldsymbol{I}_b\operatorname{diag}(\boldsymbol{\gamma})\boldsymbol{c}(n)+\boldsymbol{e}(n) \tag{2}$$

where $\boldsymbol{A}(\boldsymbol{\theta})$ is defined as $\boldsymbol{A}(\boldsymbol{\theta})\doteq\left[\boldsymbol{A}_m(\theta_0),\ldots,\boldsymbol{A}_m(\theta_K)\right]$ with $\boldsymbol{A}_M(\theta_k)\doteq\begin{bmatrix}\boldsymbol{a}_{Rc}(\theta_k) & \boldsymbol{a}_{Rx}(\theta_k)\\ \boldsymbol{a}_{Lx}(\theta_k) & \boldsymbol{a}_{Lc}(\theta_k)\end{bmatrix}$ and $\boldsymbol{\theta}\doteq[\theta_0,\ldots,\theta_K]^T$. The reflection coefficient vector and the amplitude vector are respectivel $\boldsymbol{\lambda}\doteq\left[\lambda_{R,0},\lambda_{L,0},\ldots,\lambda_{R,K},\lambda_{L,K}\right]^T$ and $\boldsymbol{\gamma}\doteq[\gamma_0,\ldots,\gamma_K]^T$, respectively. The transition matrix is $\boldsymbol{I}_b\doteq\boldsymbol{I}_{K+1}\otimes[1,1]^T$. The code waveform is $\boldsymbol{c}(n)\doteq\left[c(n-\tau_0),\ldots,c(n-\tau_K)\right]^T$.

B. Off-grid Sparse GNSS Multipath Signal Model

The spatially uniformly sampling grid points set is $\tilde{\boldsymbol{\theta}}\doteq\left[\tilde{\theta}_1,\ldots,\tilde{\theta}_{N_\theta}\right]$ with fixed angle interval r_θ . We can construct an on-grid angle-polarization over complete basis matrix $\boldsymbol{A}\doteq\left[\boldsymbol{A}_m(\tilde{\theta}_1),\ldots,\boldsymbol{A}_m(\tilde{\theta}_{N_\theta})\right]$.

To improve estimation accuracy, we assume an off-grid vector $\boldsymbol{\beta}_{\theta_s}\doteq\left[\beta_1,\ldots,\beta_{N_\theta}\right]^T$ where $\beta_{n_\theta}=\theta_k-\tilde{\theta}_{n_\theta}$ with each $\tilde{\theta}_{n_\theta}$ being the nearest grid point to the k th signal. Since the angles are the same for the LHCP and RHCP components, we define $\boldsymbol{\beta}_\theta\doteq\boldsymbol{\beta}_{\theta_s}\otimes[1,1]^T$. The off-grid angle-polarization over complete basis matrix is

$$\boldsymbol{\Phi}_\beta=\boldsymbol{A}+\boldsymbol{B}_\theta\operatorname{diag}(\boldsymbol{\beta}_\theta) \tag{3}$$

where $\boldsymbol{B}_\theta$ is a first order Taylor series expansion derivation of $\boldsymbol{A}$ with respect to $\tilde{\theta}_{n_\theta}$, and is composed of $b_\theta\left(\tilde{\theta}_{n_\theta}\right)$ where $b_\theta\left(\tilde{\theta}_{n_\theta}\right) \doteq \partial A_m\left(\tilde{\theta}_{n_\theta}\right) / \partial \tilde{\theta}_{n_\theta}$, $k = 0,\ldots,K, n_\theta = 1,\ldots,N_\theta$.

Denote $\boldsymbol{Y} \doteq \left[\boldsymbol{y}(1),\ldots,\boldsymbol{y}(N)\right]$ and $\boldsymbol{N} \doteq \left[\boldsymbol{n}(1),\ldots,\boldsymbol{n}(N)\right]$, we have a multiple snapshots model which is given by

$$\boldsymbol{Y} = \boldsymbol{\Phi}_{\boldsymbol{\beta}} \boldsymbol{X} + \boldsymbol{N} \tag{4}$$

where $\boldsymbol{X} \doteq \left[\boldsymbol{x}(1),\ldots,\boldsymbol{x}(N)\right]$ is row-sparse.

III. Joint Angle and polarization MSBL

This section derives a Bayesian method for jointly estimating from spatial and polarization domains.

A. Sparse Bayesian Formulation

Assuming elements in the white noise vector are independent and each of them has a Gaussian distribution

$$p\left(\boldsymbol{E} \middle| \sigma_n^2\right) = \prod_{n=1}^{N} \mathcal{CN}\left(\boldsymbol{e}_s(n) \middle| \boldsymbol{0}_{2M_\theta \times 1}, \sigma_n^2 \boldsymbol{I}_{2M_\theta}\right). \tag{5}$$

Define the precision $\lambda_n \doteq \sigma_n^{-2}$, a Gamma distribution is used to describe

$$p\left(\lambda_n; a, b\right) = \mathcal{B}\left(\lambda_n \middle| a, b\right) \tag{6}$$

where $\mathcal{B}\left(\lambda_n \middle| a,b\right) \doteq \left[\Gamma(a)\right]^{-1} b^a \lambda_n^{a-1} e^{-b\lambda_n}$ with the Gamma function $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$.

The data likelihood for $\boldsymbol{X}$ given $\boldsymbol{Y}$ is complex Gaussian

$$p\left(\boldsymbol{Y} \middle| \boldsymbol{X}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) = \prod_{n=1}^{N} \mathcal{CN}\left(\boldsymbol{y}(n) \middle| \boldsymbol{\Phi}_{\boldsymbol{\beta}} \boldsymbol{x}(n), \lambda_n^{-1} \boldsymbol{I}_{2M_\theta}\right). \tag{7}$$

As all the columns of $\boldsymbol{X}$ are independent and share the same prior, the prior of $\boldsymbol{X}$ is given by

$$p\left(\boldsymbol{X} \middle| \boldsymbol{\alpha}\right) = \prod_{n=1}^{N} \mathcal{CN}\left(\boldsymbol{x}(n) \middle| 0, \boldsymbol{\Upsilon}\right) \tag{8}$$

where the spatial-polarization dependent variance $\boldsymbol{\alpha} = \left[\alpha_1,\ldots,\alpha_{2N_\theta}\right]^T$ is controlling the row sparsity of $\boldsymbol{X}$ and $\boldsymbol{\Upsilon} = \mathrm{diag}(\boldsymbol{\alpha})$. A two-stage hierarchical prior is adopted

$$p(\boldsymbol{\alpha}, \rho) = \prod_{n_g=1}^{2N_\theta} \mathcal{B}\left(\boldsymbol{\alpha}(n_g) \middle| 1, \rho\right) \tag{9}$$

with $p(\boldsymbol{X}, \rho) = \int p\left(\boldsymbol{X} \middle| \boldsymbol{\alpha}\right) p(\boldsymbol{\alpha}, \rho) d\boldsymbol{\alpha}$.

For the off-grid spatial-polarized vector $\boldsymbol{\beta}_\theta$, a non-informative uniform prior is used

$$p\left(\boldsymbol{\beta}_\theta, r_\theta\right) = U\left(-\frac{1}{2} r_\theta, \frac{1}{2} r_\theta\right). \tag{10}$$

Combining the stages of the hierarchical Bayesian model, the joint distribution of $\boldsymbol{X}$ is obtained

$$\begin{aligned} & p\left(\boldsymbol{X}, \boldsymbol{Y}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) \\ & = p\left(\boldsymbol{Y} \middle| \boldsymbol{X}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) p\left(\boldsymbol{X} \middle| \boldsymbol{\alpha}\right) p(\boldsymbol{\alpha}) p(\lambda_n) p(\boldsymbol{\beta}_\theta) \end{aligned}. \tag{11}$$

B. Bayesian Inference

Using Bayesian rule, we obtain the posterior for $\boldsymbol{X}$

$$p\left(\boldsymbol{X} \middle| \boldsymbol{Y}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) \propto p\left(\boldsymbol{Y} \middle| \boldsymbol{X}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) p\left(\boldsymbol{X} \middle| \boldsymbol{\alpha}\right). \tag{12}$$

The posterior distribution of $\boldsymbol{X}$ follows the Gaussian function

$$p\left(\boldsymbol{X} \middle| \boldsymbol{Y}, \lambda_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) = \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{13}$$

with $\boldsymbol{\mu}(n) = \lambda_n \boldsymbol{\Sigma} \boldsymbol{\Phi}_{\boldsymbol{\beta}}^H \boldsymbol{y}(n)$ and $\boldsymbol{\Sigma} = \left(\lambda_n \boldsymbol{\Phi}_{\boldsymbol{\beta}}^H \boldsymbol{\Phi}_{\boldsymbol{\beta}} + \boldsymbol{\Upsilon}^{-1}\right)^{-1}$.

When calculating $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, we need to estimate λ_n, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}_\theta$. Following a similar procedure in [5] denoting $\boldsymbol{U} = \left[\boldsymbol{\mu}(1),\ldots,\boldsymbol{\mu}(N)\right]$, the updates of $\boldsymbol{\alpha}$ and λ_n are

$$\boldsymbol{\alpha}(n_g) = \left(\sqrt{N^2 + 4\rho E\left\{\left\|\boldsymbol{X}(n_g,:)\right\|_2^2\right\}} - 1\right) \Big/ 2\rho \tag{14α}$$

$$\lambda_n = \left(2M_\theta N + a - 1\right) \Big/ \left(E\left\{\left\|\boldsymbol{Y} - \boldsymbol{\Phi}_{\boldsymbol{\beta}} \boldsymbol{X}\right\|_F^2\right\} N + b\right) \tag{14β}$$

where $E\left\{\left\|\boldsymbol{X}(n_g,:)\right\|_2^2\right\} = \left\|\boldsymbol{U}(n_g,:)\right\|_2^2 + \boldsymbol{\Sigma}(n_g,n_g)$ and $E\left\{\left\|\boldsymbol{Y} - \boldsymbol{\Phi}_{\boldsymbol{\beta}} \boldsymbol{X}\right\|_F^2\right\} = \left\|\boldsymbol{Y} - \boldsymbol{\Phi}_{\boldsymbol{\beta}} \boldsymbol{U}\right\|_F^2 + \boldsymbol{\alpha}(n_g) \sum_{n=1}^{2N_\theta} \varsigma(n_g)$ with $\varsigma(n_g) = 1 - \boldsymbol{\alpha}^{-1}(n_g) \boldsymbol{\Sigma}(n_g,n_g)$, $n_g = 1,\ldots,2N_\theta$.

Maximize $E\left\{\log p\left(\boldsymbol{Y} \middle| \boldsymbol{X}, \alpha_n, \boldsymbol{\alpha}, \boldsymbol{\beta}_\theta\right) p(\boldsymbol{\beta}_\theta)\right\}$, we have

$$\hat{\boldsymbol{\beta}}_\theta = \arg \min_{\boldsymbol{\beta}_\theta \in \left[-\frac{1}{2} r_\theta, \frac{1}{2} r_\theta\right]} \left\{\boldsymbol{\beta}_\theta^T \boldsymbol{P} \boldsymbol{\beta}_\theta - 2\boldsymbol{v}^T \boldsymbol{\beta}_\theta\right\} \tag{15}$$

and

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$$\boldsymbol{P} = \Re\left\{\frac{1}{N}\sum_{n=1}^{N}\Xi^{H}\Xi\right\} + \Re\left\{\boldsymbol{\Sigma}\odot\boldsymbol{B}_{\theta}^{H}\boldsymbol{B}_{\theta}\right\}, \qquad (16\alpha)$$

$$\boldsymbol{v} = \Re\left\{\frac{1}{N}\sum_{n=1}^{N}\left\{\left(\boldsymbol{y}(n)-\boldsymbol{A\mu}(n)\right)^{H}\Xi\right\}\right\} - \Re\left\{\mathrm{diag}\left(\boldsymbol{B}_{\theta}^{H}\boldsymbol{A\Sigma}\right)^{T}\right\} \quad (16\beta)$$

with $\Xi \doteq \left(\boldsymbol{\mu}(n)^{T}\otimes\boldsymbol{I}_{M_{\theta}}\right)\left(\boldsymbol{I}_{2N_{\theta}}\otimes\boldsymbol{B}_{\theta}\right)\boldsymbol{J}_{4N_{\theta}^{2},N_{\theta}}$ and $\boldsymbol{J}_{4N_{\theta}^{2},2N_{\theta}}$ is the Khatri-Rao product of two $\boldsymbol{I}_{N_{\theta}}$.

C. Time Delay Estimation

By using the converged JAPMSBL, the beamformed signal can be acquired,

$$\hat{y}(n) = \Gamma\left(\boldsymbol{A}_{\theta}^{H} + \boldsymbol{B}_{\theta}\mathrm{diag}\left(\boldsymbol{\beta}_{\theta}\right)\right)\boldsymbol{\Sigma}^{-1}\boldsymbol{y}(n). \qquad (17)$$

The time delay of LOS signal can thus be calculated using

$$\hat{\tau}_{0} = \arg\min_{\tau_{0}}\left\|\hat{y}(n) - \frac{\hat{y}(n)^{H}c(n-\tau_{0})}{\left\|c(n-\tau_{0})\right\|^{2}}c(n-\tau_{0})\right\|. \qquad (17)$$

IV. Simulation Results

This section focuses on evaluating 1) the proposed JAPMSBL with M_{θ}-element DPA (M-DPA), 2) MSBL [1] with M_{θ}-element single-polarization antenna (M-SPA) and $2M_{\theta}$-element SPA (2M-SPA). Assume a uniform linear array with the half wavelength spacing antenna array and $M_{\theta}=4$.

One LOS and two multipath signals utmost are assumed. These signals are $\arg(\gamma_k)=0, k=0,1,2$, and the direct-to-multipath ratio is $|\gamma_0|/|\gamma_k|=3$ dB, $k=1,2$. The simulations are carried out with $C/N_0=45$ dB-Hz. We consider the GPS C/A code with code period $T=1$ ms and chip duration $T_c=977.52$ ns. We use $N=20$ ms code periods in total. We consider three signals with time delays $\boldsymbol{\tau}=[0,0.23,0.3]T_c$. The angles of LOS and the second multipath signal are 0° and 15°, respectively. As for the reflection coefficients, we assume $\arg(\lambda_{R,k})=0.1\pi, k=1,2$ and $\arg(\lambda_{L,k})=-0.7\pi, k=1,2$, and values $|\lambda_{R,k}|=|\lambda_{L,k}|=0.7$. Considering mutual coupling effects of DPA, we model $\boldsymbol{a}_{Rc}(\theta_k)=\boldsymbol{a}_{Lc}(\theta_k)=\boldsymbol{a}(\theta_k)$, $\boldsymbol{a}_{Rx}(\theta_k)=\kappa_R\boldsymbol{a}(\theta_k)$, and $\boldsymbol{a}_{Lx}(\theta_k)=\kappa_L\boldsymbol{a}(\theta_k)$ where $\boldsymbol{a}(\theta_k)$ is the steering vector of an uniformly distributed antenna. The cross-talk parameters $\kappa_R=\kappa_L=-10$dB are chosen.The scanning angle grid is uniformly distributed in the range from -90° to 90° with angle interval of $r_{\theta}=2°$. In addition, the hyperparameters are $\rho=1e^{-2}$ and $a=b=1e^{-4}$.

Varying from angle difference between the LOS and the first multipath signal, the RMSEs of LOS time delay are acquired.

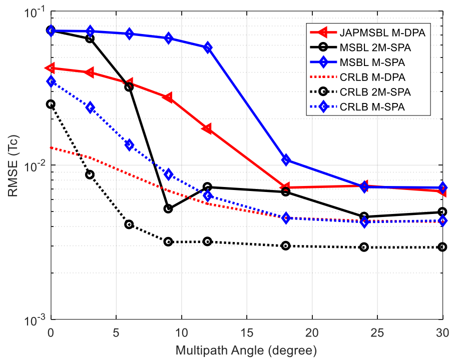

Fig. 1 RMSEs of LOS time delay via varied angle difference in case of 2 rays

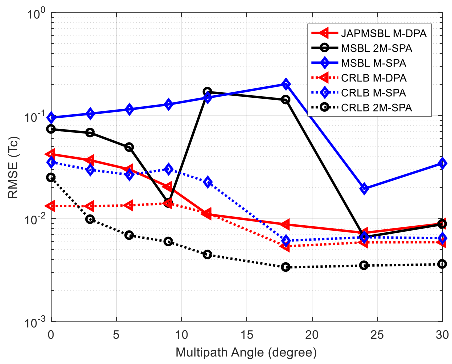

Fig. 2 RMSEs of LOS time delay via varied angle difference in case of 3 rays

The RMSE of MSBL with 2M-SPA, as detailed in Fig. 1, generally behaves better than that of JAPMSBL with M-DPA across large angle differences. JAPMSBL with M-DPA substantially outperforms the other methods from 0° to 5°.

In the scenario where three rays exhibit high correlations in space (see in Fig. 2), MSBL with M-SPA results in limited improvement. However, JAPMSBL significantly improves and yields results closer to the CRLB [2].

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